УДК 621.3
DOI: 10.31732/2663-2209-2020-60-142-156

ВИКОРИСТАННЯ ДАНИХ ПРО ВАРТІСТЬ ЖИТТЯ ДЛЯ ПОРІВНЯННЯ МІЖ МЕТОДОМ МАКСИМАЛЬНОЇ ВІРОГІДНОСТІ ТА МЕТОДОМ МОМЕНТУ ДЛЯ ОЦІНКИ ПАРАМЕТРІВ РОЗПОДІЛУ ГАММИ

Таха Мохамед Дааб
аспірант, ВНЗ «Університет економіки та права «КРОК», м. Київ, вул. Табірна, 30-32, 03113, Україна, тел.: (044)-455-69-82, e-mail: tahadaab@gmial.com, ORCID: https://orcid.org/0000-0001-6761-964X

USING A COST OF LIVING DATA FOR A COMPARISON BETWEEN MAXIMUM LIKELIHOOD METHOD AND MOMENT METHOD FOR ESTIMATING GAMMA DISTRIBUTION PARAMETERS

Taha Mohamed Daab
postgraduate student, "KROK" University, Kyiv, st. Tabirna, 30-32, 03113, Ukraine, tel.: (044)-455-69-82, e-mail: tahadaab@gmial.com, ORCID: https://orcid.org/0000-0001-6761-964X

Анотація. *Широке використання методів теорії ймовірностей і математичної статистики в економічній науці і практиці вимагає більш широкої та ґрунтовної підготовки спеціалістів різних галузей економіки в царині освоєння ними сучасних математичних знань. Практично всі процеси, що відбуваються в організації виробництва, фінансах, маркетингу, банківській справі, менеджменті, мають елементи невизначеності, складності, багатопричинності, тобто характеризуються випадковістю. Тому вельми важливим для керування економікою є встановлення закономірностей у випадкових явищах. Використання статистичних методів обробки даних дає змогу виявити реальні закономірності, які об'єктивно існують у масових випадкових явищах. На основі такої обробки даних можна з певною точністю робити прогноз розвитку економіки, обчислювати ризики економічної діяльності, передбачувати кризи та інші соціально-економічні явища, які мають випадковий характер. В статті проаналізовано вивчення вартості життя, адже для їх вираження іноді потрібно вивчити більшість явищ, виражених випадковою величиною, значенням якої є населення, функція ймовірності якого має обмежену математичну формулу, але це залежить від за деякими невідомими параметрами і значення чи значення яких визначаються, розподіл повністю визначається пошуком необхідністі методів, які можуть бути використані для визначення невідомі параметри населення, за допомогою яких відбирається випадкова вибірка з цієї сукупності. За допомогою цих зразкових можна знайти функцію, яка використовується як інструмент для визначення значення . Ця функція називається оцінювачем. У нашому дослідженні ми розглянемо найважливіші методи оцінки параметрів за допомогою даних про вартість життя, які є двома методами "Максимальна ймовірність" та "Імовірність ймовірності", та їх використання при оцінці параметрів " Розподіл гамми ". Їх порівнювали, щоб з'ясувати, який метод кращий для оцінки параметрів.*

Ключові слова: *вартість життя; популяції; функція ймовірності; оцінювач; розподіл гамми; метод; параметри; максимальна ймовірність; імпульсна ймовірність; статистичне населення.*
Формули: 20; рис.: 10; табл.: 5, бібл.: 5.

Annotation. *Widespread use of methods of probability theory and mathematical statistics in economics and practice requires a broader and more thorough training of specialists in various sectors of the economy in the field of their development of modern mathematical knowledge. Virtually all processes occurring in the organization of production, finance, marketing, banking, management, have elements of uncertainty, complexity, multiplicity, ie are characterized by chance. Therefore, it is very important to manage the economy is to establish patterns in random phenomena. The use of statistical methods of data processing makes it possible to identify real patterns that objectively exist in mass random phenomena. On the basis of such data processing it is possible to make a forecast of economic development with certain accuracy, to calculate the risks of economic activity, to predict crises and other socio-economic phenomena that are random. When we studying data related to the cost of living Since its expression sometimes requires studying most of the phenomena of " Populations " expressed by a random variable (X), whose value is a Population whose probability function* $f(x,\theta)$ *has a limited mathematical formula, but it depends on some unknown parameters "*θ*" and whose value or values are determined the distribution is completely determined and thus [all its commas are determined] and this is why it called The need to search for methods that can be used to determine the "Estimation" of the unknown Population parameters through which a random sample is drawn from that population.*

Using this sample data $t(x\ ,x_2.......x_n)$ *a function can be found in the sample data* $t(x\ ,x_2.......x_n)$ *that is used as a tool for determining a value "* θ *". This function is called the Estimator. In our study, we will deal with the most important methods for Estimating the parameters by using the cost of living data, which are the two methods of "Maximum likelihood " and " Momentum likelihood ", and their use in Estimating the parameters of the "Gamma Distribution ". They were compared to find out which method is better for Estimating the parameters.*

Keywords: *cost of living, Populations, probability function, Estimator, Gamma Distribution, method, Parameters, Maximum likelihood, Momentum likelihood, statistical Population.*

Formulas: 20; fig.: 10; tabl.: 5;bibl.: 5.

Introduction. When presenting data of the cost of living and methods of collecting them for use in statistical analysis, especially in estimating some statistical parameters and estimation theory, Estimation theory in addition to hypothesis tests is one of the most important statistical inference rules, and this importance comes from our constant need to explain the phenomena surrounding us and define their distinctive features and characteristics in convincing statistical methods, and it is the method by which we conclude Population, where we are interested in estimating the "Parameters".Unknowns of the probability distributions describing the results of a randomized trial. This is because most of these phenomena can be represented by a certain probabilistic model that highlights their most important characteristics after defining a random variable for this phenomenon, but this distribution depends on an unknown parameter or unknown features, and these parameters must be searched for an accurate scientific method to estimate the best possible value for it based on what We have available information in the random sample drawn to study this phenomenon.

The concept of " Estimating " is used in many areas in our daily life, where the traveler estimates the time he needs to travel a certain distance, or the head of the family to estimate the monthly family expenses, and the estimate includes in its general form a combination of interconnected vocabulary such as:

Estimating: It is a method based on the calculations of some statistics from the sample data that give estimated values for the corresponding parameters in the statistical population chosen from it.

Estimator: A statistic that determines how sample data is used to estimate the Unknowns population parameter.

Literature Review. There is a lot of research and studies presented on this topic, for example, the book of statistical inference "The Theory of Estimation" by Professor "Abdelhafid Muhammad Fawzi Mustafa", and also, "Professor. Ali Abdul Salam Al-Ammari ", and " Professor. Ali Hussain Al-Ajili ", in the book of Statistics Theoretical Probabilities and Application, and through many research papers and publications that cover this field, and of course it cannot be limited.

But through this paper, I will use the maximum likelihood method and moment method to estimate the parameters of the "gamma distribution" after generating a different random sample, to study the results of the analysis to reach the best estimation model recommended to use.

Because Sometimes the application of differentiation methods to determine the estimations of the parameters may not lead to a realistic result, especially if the scope of the variable depends on the unknown distribution parameters

Aims. The main objective of this paper by using the cost of living data and using estimation is the need to know some characteristics of the " statistical Population " represented in the parameters such as " mean-variance ", the ratio of the elements of the" statistical Population" that bear a certain characteristic as long as this is the case, two reasons are justifying the use of estimation methods:

The many societies of importance, although they are limited but large, and therefore examining all their vocabulary is not possible in terms of costs.

There are infinite or infinite "Populations" and thus examining all their vocabulary is not possible.

Hence… any statistic used to estimate a function $\tau(\theta)$ is known as an "estimator" for the function $\tau(\theta)$, which is often denoted by a symbol $T = t(x \ , x_2 ....x_n)$.

After obtaining, collecting, and organizing the cost of living data We provide an overview of Estimation theory, it's the branch of statistics and signal processing that deals with estimating values from Parameters based on data that contain a random component. Parameters describe an underlying physical condition in such a way that their values influence the distribution of the measured data. As the Estimator attempts to approximate values of unknown parameters using measurements. This estimate is based on a random sample of different sizes that are generated. The data is assumed to be random with a probability distribution dependent on the parameters of interest.

Therefore, we will review the estimation methods and how to use them to find an estimate for the parameters of the distribution, and also study the results of the analysis to arrive at the methods of estimation that are recommended in the statistical analysis.

Results. In my paper by studying the estimation theory and the most common methods used the cost of living data to estimate the parameters of the distributions, and after generating a random sample of different sizes, belong to gamma distribution to estimate its parameters each time, and after performing the statistical analysis process by the method of maximum likelihood method (MLE) and the method of moment likelihood method (MME) to estimate the parameters of the distribution and studying in the results that give us the general shape or not.

And through it, we found that the maximum likelihood method (MLE) gives results that are close to the parameters of the original study from the moment method (MME), what has been proven in the study, and we recommend the maximum likelihood method (MLE) to estimate the statistical parameters.

Point estimation:

It is said that the estimator is valued at one value for the parameter of the "statistical Population" if one numerical value is given as an estimate for that parameter, and it seems clear from the definition of the estimate that it is a single choice $\left(x_1 ,.x_{2,} , x_3 ....x_n\right)$, It has a probability cumulative or density that depends on the parameter θ and we denote this function with the symbol $f(x,\theta)$, because to obtain information on the distribution of this variable, and in particular on the value of the parameter θ, we need to take a sample from this distribution and use it to obtain an estimator for a single value for the parameter θ, and in some applications we will find that Estimates can be obtained using what corresponds to it in the sample, for example, to estimate the population means we will use the sample mean (μ), and to estimate the percentage of elements or items of the statistical community that have a specific characteristic or characteristic (P) we will use the sample percentage .... etc.

It is used as an estimator for the unknown parameter. The comparison between the different estimators depends on:

1. How to determine the statistic $t(X \ , X_2 ......X_n)$ that can be used as an estimator for the unknown parameter.
2. How to differentiate between the different capabilities.

Methods of finding estimators:

Assuming that $(X_1 ,.X_{2,} , X_3 ....X_n)$ a random sample and the probability density function a random $f(x,\theta)$ and it contains the unknown parameter θ, and also assuming that θ it belongs to the category of real numbers $\tilde{\theta} = (\theta_1 , \theta_2 ....\theta_k)$. Thus, it is possible to define the set of all real numbers that this parameter takes θ. It is called "parameter space" and denoted by the symbol (Ω), The goal is to find statistics that are used to estimate unknown parameters $\theta_i \, ; i = 1,2,3,...k$, and there are many ways to estimate these parameters, the most important of which is the "method of moments" attributed to" Karl Pearson", and the "method of maximum likelihood" attributed to the scientist "R. A. Fisher".

Most of the phenomena "populations" can be expressed by the random variable (X), whose probability function $f(x,\theta)$ has a specific mathematical formula, but it depends on some unknown parameters θ and by determining its value or values the distribution is determined completely [and thus all its properties are determined], so it is called The need to search for methods that can be used to determine the "estimation" of the values of the unknown pupation parameters that are made by drawing a sample $t(X\ ,X_2.......X_n)$ from that population and using the sample data can find a function θ , The function is called $t(X_1,X_2,...Xn)$ Estimator while it is called $t(x\ ,x_2.......x_n)$ a value of estimation, and it is the value of the estimator after knowing the sample values, and this is what we will explain.

Now the question is how to obtain an estimate of one value for the unknown parameter ??? And the answer to this question there are several methods used to obtain an estimate of one value for the unknown parameter, for example :

* maximum likelihood function
* least-squares method
* method of moments
* Baise method

Maximum likelihood function:

It is symbolized by the symbol [MLE], an abbreviation of its name consisting of the initials of each term, and this method is considered one of the most important methods of estimation, and the most used one, and it depends in the research on the capabilities of the unknown parameters of the population on the likelihood function. To simplify this item, we will detail it as follows:

Likelihood Function: If we assume that (X) a random variable of a "population distribution" has a mass or density function $f(x,\theta)$ Where the θ parameter of the distribution is unknown and was $(x_1,x_2,....x_n)$ A random sample of number (n) is represented from this distribution, so the probability function for this sample is as follows:

$$l(\theta) = f(x_1,\theta)f(x_2,\theta)f(x_3,\theta)...................... \quad (1)$$

If the variables are independent then:

$$l(\theta) = \prod_{i=1}^{n} f(x_i,\theta) \quad (2)$$

The Maximum likelihood MLE:

The magnitude of the Maximum-likelihood of the features of the distribution is the feature values that make the weight function $L(\theta)$ the largest possible.

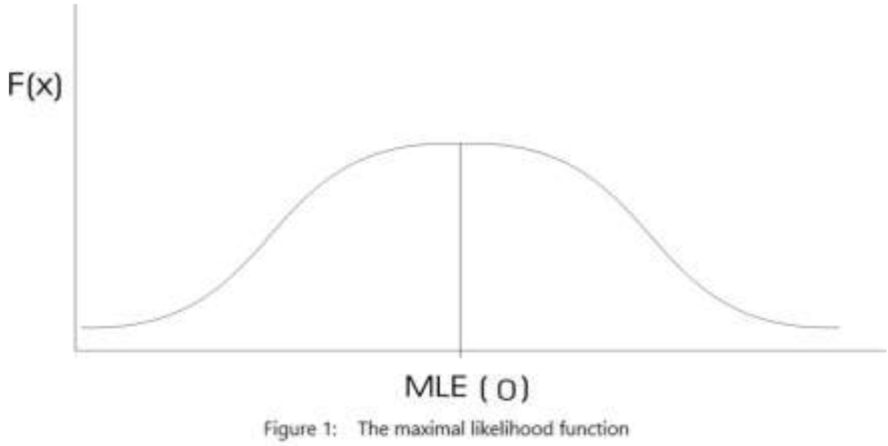

Figure 1. The maximal likelihood function

We obtain an estimate for the parameter θ by solving the following equation:

$$\frac{\partial l}{\partial \theta} = \frac{\partial l(\theta, x)}{\partial \theta} = 0 \Rightarrow$$

$$\frac{\partial \ln l}{\partial \theta} = \frac{\partial \ln l(\theta, x)}{\partial \theta} = 0 \qquad (3)$$

Where:

$$\frac{\partial^2 \ln l}{\partial \theta^2} \prec 0 \qquad , \frac{\partial^2 l(\theta, x)}{\partial \theta^2} \prec 0 \qquad (4)$$

However… it must be noted that there are some non-differential likelihood functions such as the uniform distribution $\cup(0,\theta)$ and in general, the distributions in which the extent of the random variable depends on the parameter are not subject to differentiation, and in this case, the value of the parameter "estimate" is found that makes the weighting function greater than possible. By logical discussion, drawing, and clarification, this value represents the maximum likelihood estimate.

General properties of the Maximum likelihood estimator:

1- If the total estimate $T = t(x)$ for the parameter θ can be found, then the maximum likelihood estimator is always in terms of this sufficient estimator.
2- If a complete sufficient estimator can be found, then the greatest likelihood estimator is also indicative.
3- If θ it is an unbiased estimator and its variance is the minimum [Kramer-Rao] that is:

$$\mathrm{E}(\hat{\theta}) = \theta, V(\hat{\theta}) = C.R.L.B \qquad (5)$$

This estimator is the highest likelihood estimator, but the opposite is not necessarily always correct, that is, it is not necessary that the greatest likelihood estimator is always unbiased and a variance applies to the minimum [Kramer-Rao].
4- The property of stability and unbiased, i.e. if any $T = \hat{\theta}$ The parameter of maximum likelihood of the is estimated θ It is $h(\hat{\theta}) = h(\theta)$ The maximum likelihood of the function is estimated $h(T)$. 5- If $T = \hat{\theta}$ it is the highest likelihood estimator, then: $T \approx N(\theta, C.R.L.B)$.
That is, the distribution is rough of a normal distribution and differentiated is (C. R. L.B) and that is whenever the sample size is large.
6- The high likelihood estimator does not always have to be unbiased.

Method of Moments MME :

Sample momentum is considered one of the good options as a beginning to search for the capabilities of the parameters of the population. This method has been shown based on the visual momentum of the sample. For sample and population

The steps for determining the torque capabilities of the unknown parameters can be summarized as follows:
1- Determines the resolve of population $(Mr, M'r)$.

2- Determines the torque of the sample $(mr, m'r)$.

3- Equalizing the resonant resolve of the sample with the resonant resolve of population, i.e: Number of population milestones $Mr = mr \quad M'r = m'r$ r=1.2.3...

That is to estimate *the* k parameter, you need a k equation in the previous image, noting that the population moment must function in the unknown parameters, and if a population moment is found that is not a function of the unknown parameters, it will be replaced by the next moment.

Gamma Distribution:

The picture of the probability function is as follows:

$$x \approx f(x,\alpha,\beta) = \frac{\beta^{\alpha}}{\rceil\alpha} x^{\alpha-1} e^{\beta x}; x \succ 0, \alpha \succ 0, \beta \succ 0. \qquad (6)$$

$$E(X) = \frac{\alpha}{\beta}, \qquad Ver(X) = \frac{\alpha}{\beta^2} \qquad (7)$$

Estimating Gamma distribution parameters (α, β) by moment method:

If we take a random sample from the Gamma distribution whose parameters (α, β) we have and want to estimate them by moment method (MME), then we use the following method:

$$x \approx f(x,\alpha,\beta) = \frac{\beta^{\alpha}}{\rceil\alpha} x^{\alpha-1} e^{\beta x}; x \succ 0, \alpha \succ 0, \beta \succ 0. \qquad (8)$$

We define moments as follows:

$$\grave{m}_1 = M_1' \quad \text{and} \quad \grave{m}_2 = M_2' \qquad (9)$$

$$m_1' = \overline{X} \Leftarrow \quad \text{and} \quad m_2' = \overline{X}^2 \qquad (10)$$

Whereas:

$$E(X) = \overline{X} = \frac{\alpha}{\beta} \quad \text{and} \quad \bar{X^2} = \frac{\alpha(\alpha+1)}{\beta^2} \qquad (11)$$

By solving the two equations simultaneously, we get:

$$\bar{X^2} = \left(\frac{\alpha}{\beta}\right)^2 + \frac{\alpha}{\beta}\left(\frac{1}{\beta}\right) = \qquad (12)$$

$$\Rightarrow \bar{X^2} = (\overline{X})^2 + \overline{X}\left(\frac{1}{\beta}\right)$$

$$\frac{1}{\beta} = \frac{\bar{X^2} - \overline{X}^2}{\overline{X}} \Rightarrow \beta = \frac{\overline{X}}{\bar{X^2} - \overline{X}^2} \qquad (13)$$

$$\therefore MME(\alpha) = \hat{\alpha} = \hat{\beta}\overline{X} \Rightarrow \frac{\bar{X^2} - \overline{X}^2}{\overline{X}} = \frac{\bar{X^2}}{S^2}$$

$$\Rightarrow MME(\beta) = \hat{\beta} = \frac{\overline{X}}{\bar{X^2} - \overline{X}^2}$$

Whereas:

$$\overline{X}^2 = \left(\frac{\sum_{I=1}^{N} x_i}{n}\right)^2 ; \overline{X}^2 = \frac{\sum_{i=1}^{n} x_i^{\;2}}{n} \qquad (14)$$

Estimation Gamma distribution function parameters (α, β) by maximum likelihood method:

If we have a random sample of the gamma distribution by its two parameters (α, β) and we want to estimate it using the maximum likelihood method, we have to follow the following:

$$x \approx f(x,\alpha,\beta) = \frac{\beta^\alpha}{\rceil\alpha} x^{\alpha-1} e^{\beta x}; x \succ 0, \alpha \succ 0, \beta \succ 0. \qquad (15)$$

$$L(\alpha,\beta) = \prod_{i=1}^{n} f(x;\alpha,\beta) = \prod_{i=1}^{n}\left(\frac{\beta^\alpha}{\rceil\alpha} x^{\alpha-1} e^{-\beta x}\right)$$

$$= \beta^{nr}\left[\rceil\alpha\right]^{-n}\left(\prod_{i=1}^{n} x\right)^{\alpha-1} e^{-\beta\sum x}$$

$$\Rightarrow \ln L(\alpha,\beta) = n\alpha\ln\beta - n\ln\rceil\alpha + (\alpha+1)\sum_{i=1}^{n}\ln x - \beta\sum x \qquad (16)$$

Since there are two unknown parameters, we need two simultaneous equations:

$$\frac{\partial}{\partial\alpha} L(\alpha,\beta) - \frac{n\frac{d}{d\alpha}\rceil\alpha}{\rceil\alpha} + \sum \ln x$$

$$\because \frac{d}{d\alpha}\ln(\alpha,\beta) = 0, \frac{d}{d\beta}\ln(\alpha,\beta) = 0 \Rightarrow$$

$$\frac{d}{d\beta}\ln(\alpha,\beta) = \frac{n\alpha}{\beta} - \sum x = 0$$

$$n\ln\beta - \frac{n\frac{d}{d\alpha}\rceil\alpha}{\rceil\alpha} + \sum \ln x = 0 \qquad (17)$$

To obtain the probability estimates, we must solve the two equations simultaneously.

Due to the dif $\frac{d}{d\beta}\ln(\alpha,\beta) = 0$ ficulty of determining $(\frac{d}{d\alpha}\rceil\alpha)$ within it, it is difficult to determine the probability of the likelihood of each of (α, β) the explicit images, and it is possible to resort to numerical methods to determine the value of the two equations simultaneously.

It is noticeable that if (α) it is known, then the probability estimator is obtained by solving the equation Which is as follows :

$$\frac{n\alpha}{\beta} - \sum x = 0 \Rightarrow \beta = \frac{n\alpha}{\sum x}, \hat{\beta} = \frac{\alpha}{\overline{X}}$$

$$\Rightarrow MLE(\beta) = \hat{\beta} = \frac{\alpha}{\overline{X}} \qquad (18)$$

If (β) it is known, then the estimate of the likelihood for (α) is obtained by solving the equation:

$$n\ln\beta - n\frac{\frac{d}{d\alpha}\rceil\alpha}{\rceil\beta} + \sum \ln x = 0 \qquad (19)$$

$$\frac{d}{d\alpha}\ln(\alpha,\beta) = 0 \qquad (20)$$

The solution to this equation can only be done through numerical methods using determining a value of/that satisfies this equation, so it is the probability estimator for the parameter (α).

Sometimes the application of differentiation methods to determine the estimations of the distribution parameters may not lead to a realistic result, especially if

<u>the scope of the variable depends on the unknown distribution parameters</u>

To apply the theory, where 1000 random samples were generated of different sizes, that samples followed the Gamma distribution with its two parameters (α, β), were generated. And we assumed that : [$(\beta = 0.5)$, $(\alpha = 0.2)$] and [$(\beta = 2)$, $(\alpha = 1)$] and [$(\beta = 7)$, $(\alpha = 5)$] and [$(\beta = 2)$, $(\alpha = 0.2)$] and [$(\beta = 0.2)$, $(\alpha = 2)$] to compare between the results of estimate and which are close to the parameters by using the [Moment likelihood method MME] and the [maximum likelihood method MLE], and The results of the estimate were as follows:

Table 1

We assumed that [$(\beta = 0.5)$, $(\alpha = 0.2)$]

N	MME ALFA	MLE ALFA	MME BETA	MLE BETA
10	0.279520	0.257201	1.27952	0.311702
20	0.474886	0.252272	1.47489	0.114444
30	0.471524	0.142764	1.47152	0.272840
40	0.274843	0.403697	1.27484	0.281023
50	0.254407	0.274543	1.25441	0.310502
100	0.222409	0.310896	1.22241	0.389618
150	0.158670	0.265492	1.15867	0.375218
200	0.190567	0.203718	1.19057	0.375264
250	0.178153	0.181359	1.17815	0.557193
300	0.250902	0.266257	1.25090	0.618380
350	0.245116	0.210345	1.24512	0.372364
400	0.232743	0.189542	1.23274	0.437168
450	0.231083	0.245676	1.23108	0.470625
500	0.193835	0.185713	1.19384	0.427252
550	0.198905	0.224020	1.19890	0.500054
600	0.190084	0.248735	1.19008	0.423486
650	0.139306	0.196319	1.13931	0.484098
700	0.141193	0.185265	1.14119	0.555087
750	0.213744	0.208581	1.21374	0.481392
1000	0.219609	0.222212	1.21961	0.497269

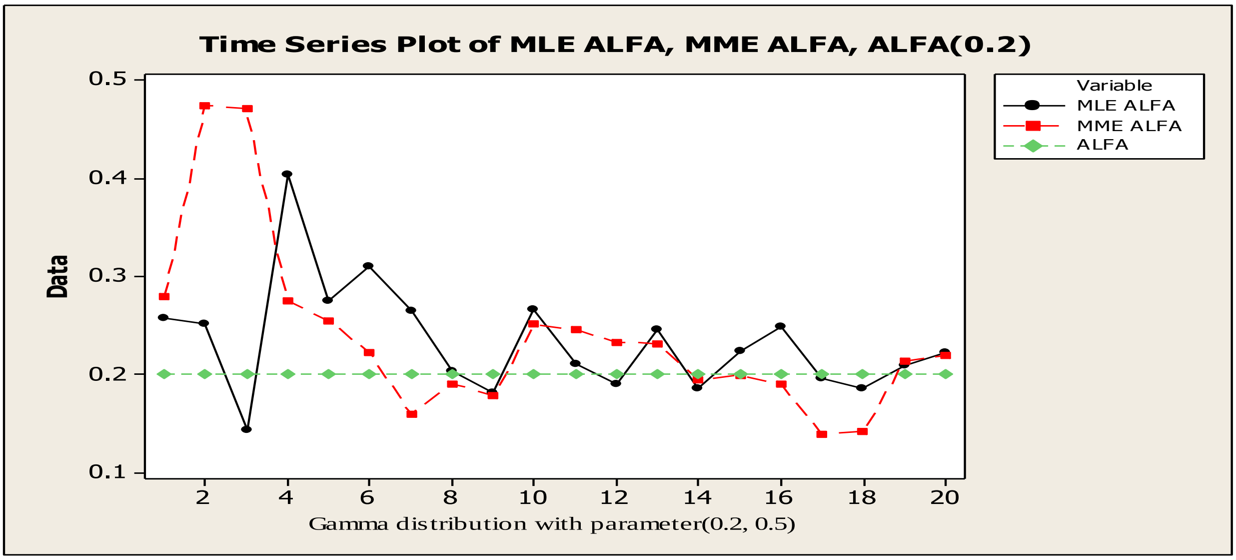

Figure 1. Gamma distribution with parameters [0.2, 0.5] by the cost of living data.

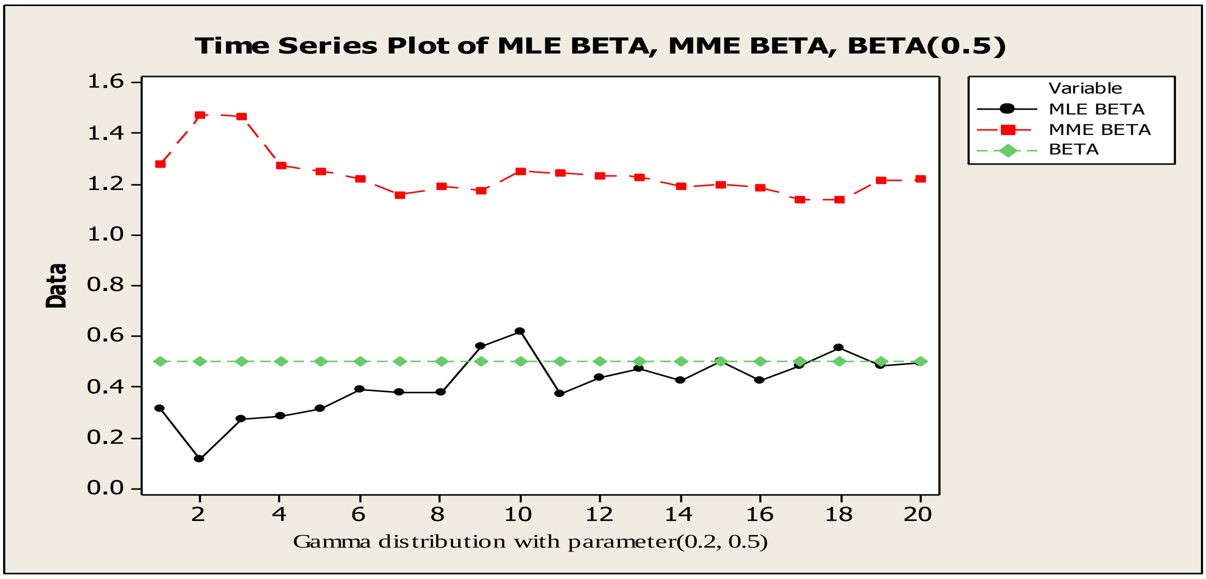

Figure 2: Gamma distribution with parameters [0.2, 0.5] by the cost of living data

Table 2

We assumed that [$(\beta = 2)$, $(\alpha = 1)$]

N	MME ALFA	MLE ALFA	MME BETA	MLE BETA
10	3,44544	0.023227	4,44544	1.492118
20	2,19535	0.972749	3,19535	2.054048
30	1,28240	1.043997	2,28240	2.490966
40	1,06090	0.929654	2,06090	2.128308
50	0,91949	1.039074	1,91949	1.549047
100	1,29147	1.151232	2,29147	1.656400
150	1,19800	1.226613	2,19800	1.344294
200	0,93915	0.841850	1,93915	2.397500
250	1,09242	0.852460	2,09242	2.377272
300	1,06456	1.109617	2,06456	1.837761
350	0,96091	1.037947	1,96091	1.986891
400	0,93560	0.972708	1,93560	2.002037
450	1,05125	0.865107	2,05125	2.166369
500	1,07963	0.845050	2,07963	2.320707
550	1,08579	1.174116	2,08579	1.698697
600	0,85307	1.113873	1,85307	1.901347
650	1,12329	0.936881	2,12329	2.155671
700	1,00135	1.102879	2,00135	1.863764
750	1,00997	0.911307	2,00997	2.248549
1000	1.21607	1.048636	0.21607.	1.970910

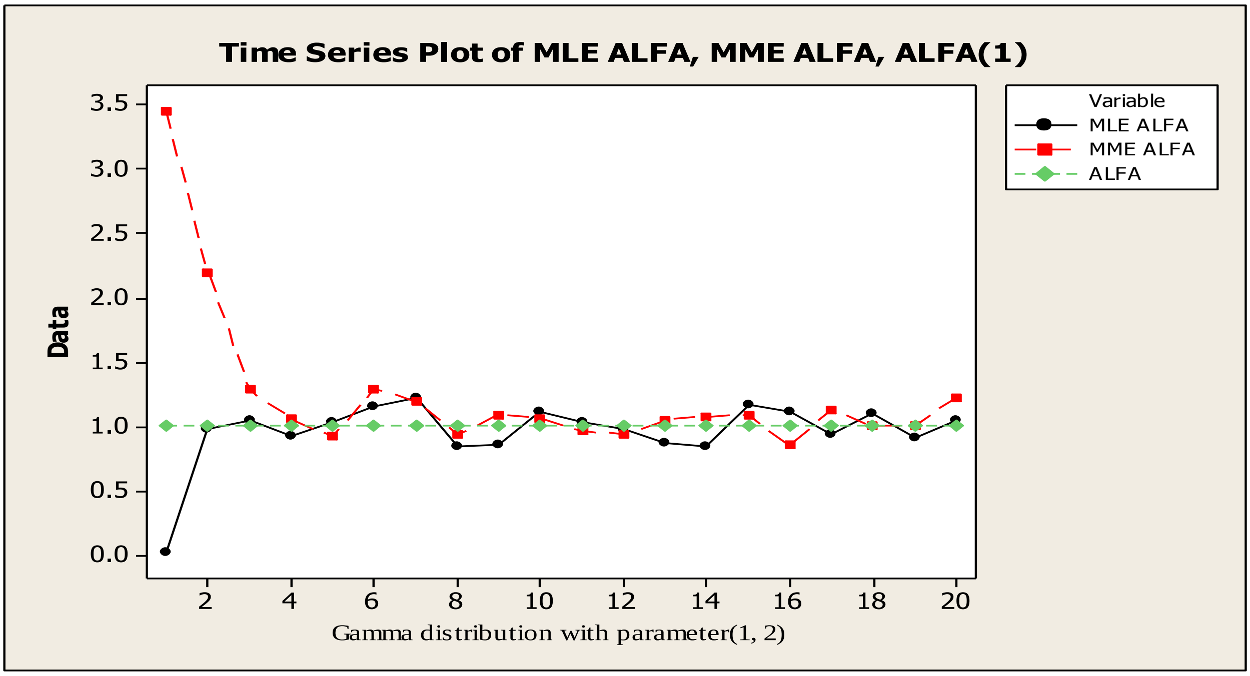

Figure 3. Gamma distribution with parameters [1, 2] by the cost of living data

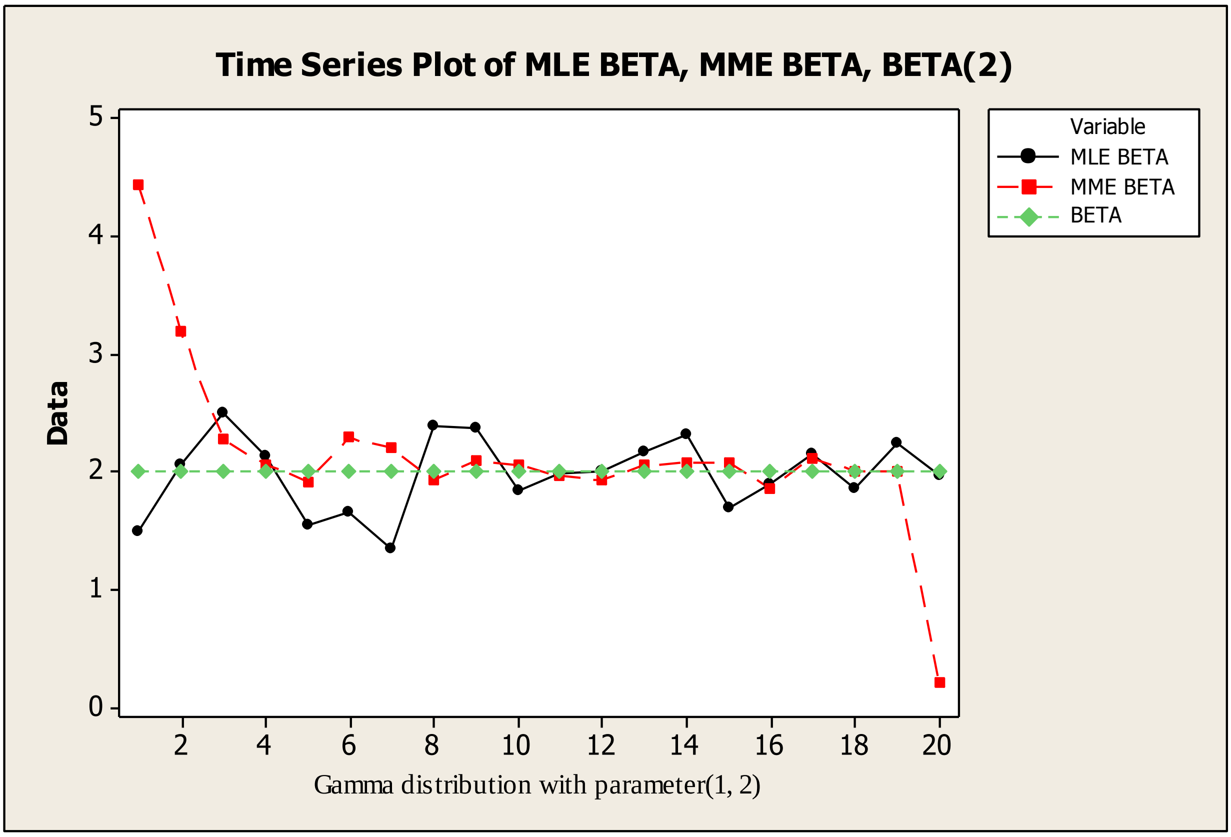

Figure 4: Gamma distribution with parameters [1, 2] by the cost of living data

Table 3

We assumed that and [$(\beta = 7)$, $(\alpha = 5)$]:

N	MME ALFA	MLE ALFA	MME BETA	MLE BETA
10	3,63604	3.070728	4,63604	10.087898
20	5,02180	3.266133	4,63604	9.103583
30	4,44806	4.850332	4,63604	7.989014
40	6,84910	4.065619	7,84910	9.187471
50	5,51495	3.936229	6,51495	9.325596
100	5,09383	5.807081	6,09383	5.819205
150	5,18216	4.590911	6,18216	7.741206
200	5,41982	4.787428	6,41982	7.040766
250	5,19097	5.004776	6,19097	7.239905
300	5,02357	4.418061	6,02357	8.020335
350	4,59697	4.905704	5,59697	6.976649
400	5,01819	5.100841	6,01819	6.988109
450	5,13005	4.607434	6,13005	7.801847
500	4,72134	4.872124	5,72134	7.188714
550	4,72134	4.514805	6,59859	7.821228
600	4,89940	4.856568	5,89940	7.239364
650	4,95518	4.747724	5,95518	7.276618
700	4,85168	5.444349	5,85168	6.440684
750	4,80788	5.674870	5,80788	6.098952
1000	0,08920	5.242401	1,08920	6.591257

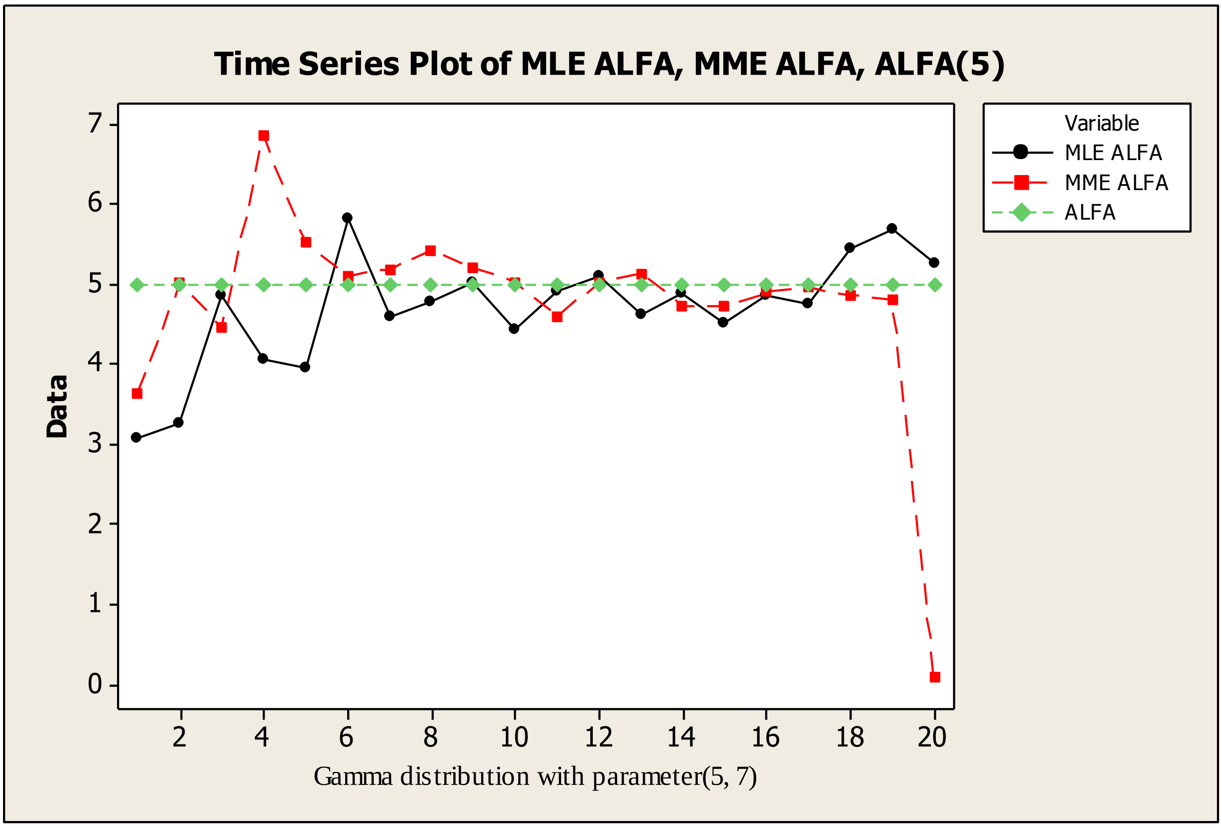

Figure 5: Gamma distribution with parameters [5, 7] by the cost of living data

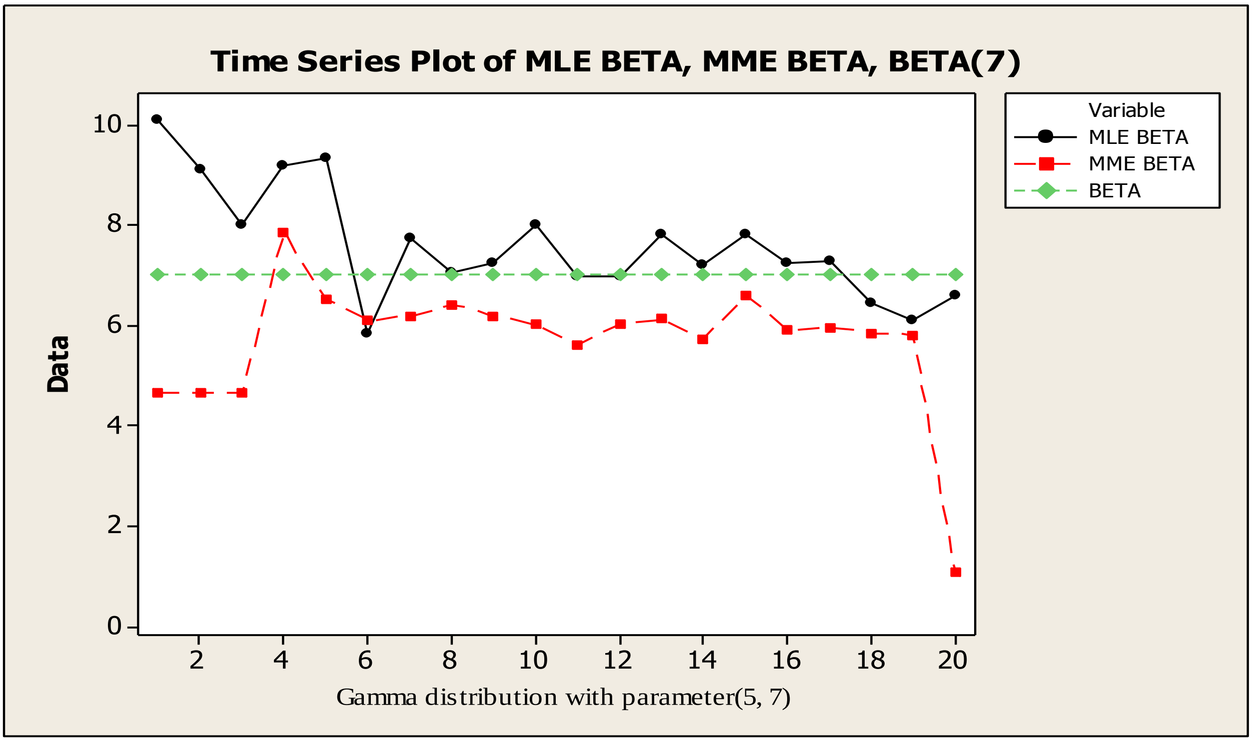

Figure 6. Gamma distribution with parameters [5, 7] by the cost of living data

Table 4

We assumed that and [$(\beta = 2)$, $(\alpha = 0.2)$] :

N	MME ALFA	MLE ALFA	MME BETA	MLE BETA
10	0.264742	0.535721	1.26474	1.304691
20	0.431913	0.368820	1.43191	0.556095
30	0.258247	0.161856	1.25825	2.613269
40	0.226633	0.228432	1.22663	1.207496
50	0.129017	0.166138	1.12902	2.335811
100	0.238494	0.132509	1.23849	3.565307
150	0.206524	0.161410	1.20652	1.922598
200	0.246978	0.265820	1.24698	1.860057
250	0.249645	0.211528	1.24965	1.646600
300	0.268712	0.22675	1.26871	1.8857792
350	0.209453	0.147798	1.20945	2.425844
400	0.169771	0.209310	1.20945	2.131580
450	0.213685	0.209454	1.20945	1.879618
500	0.243185	0.204804	1.24318	1.867427
550	0.171357	0.195839	1.17136	2.007980
600	0.211885	0.245263	1.21189	1.600985
650	0.193987	0.223579	1.19399	2.086347
700	0.217576	0.148507	1.19399	2.760141
750	0.196201	0.209775	1.19620	2.029039
1000	0.203025	0.186116	1.20302	2.212099

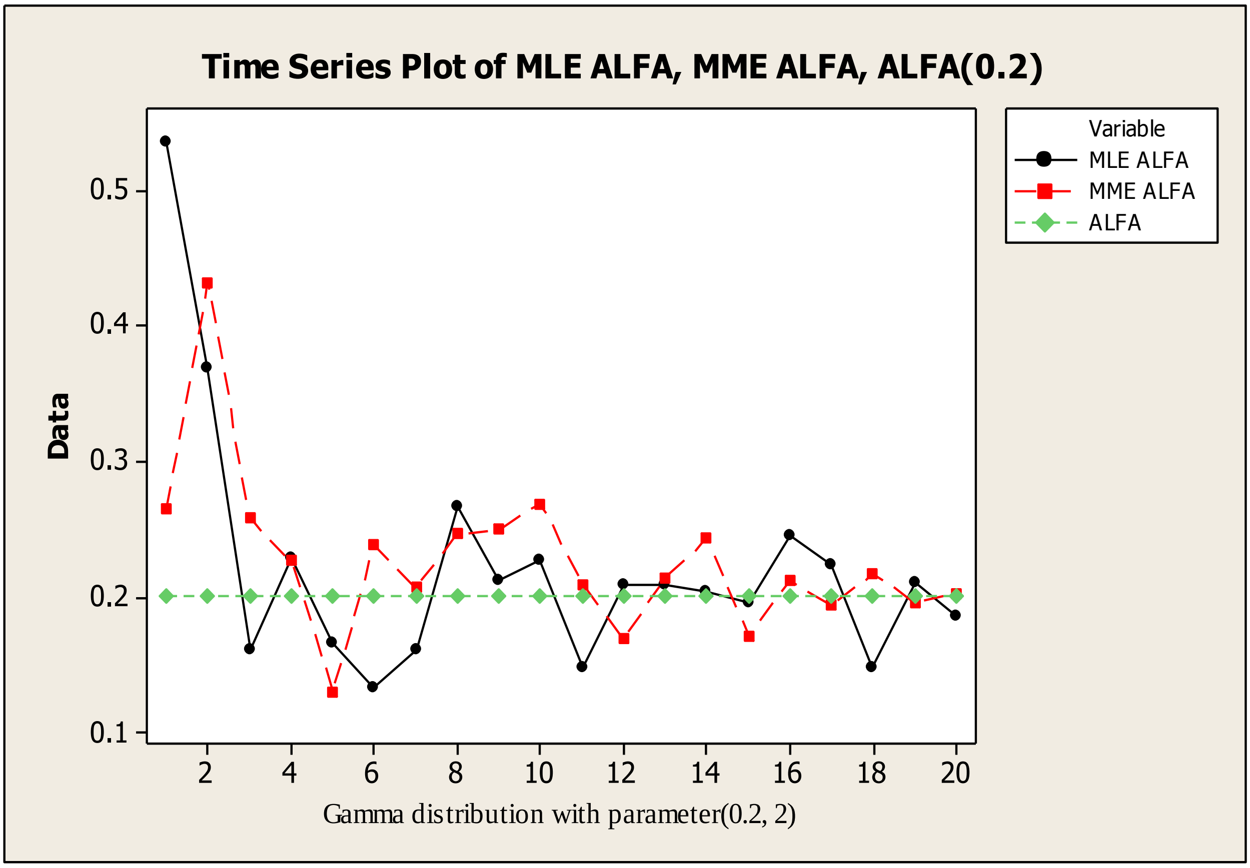

Figure 7. Gamma distribution with parameters [0.2, 2] by the cost of living data

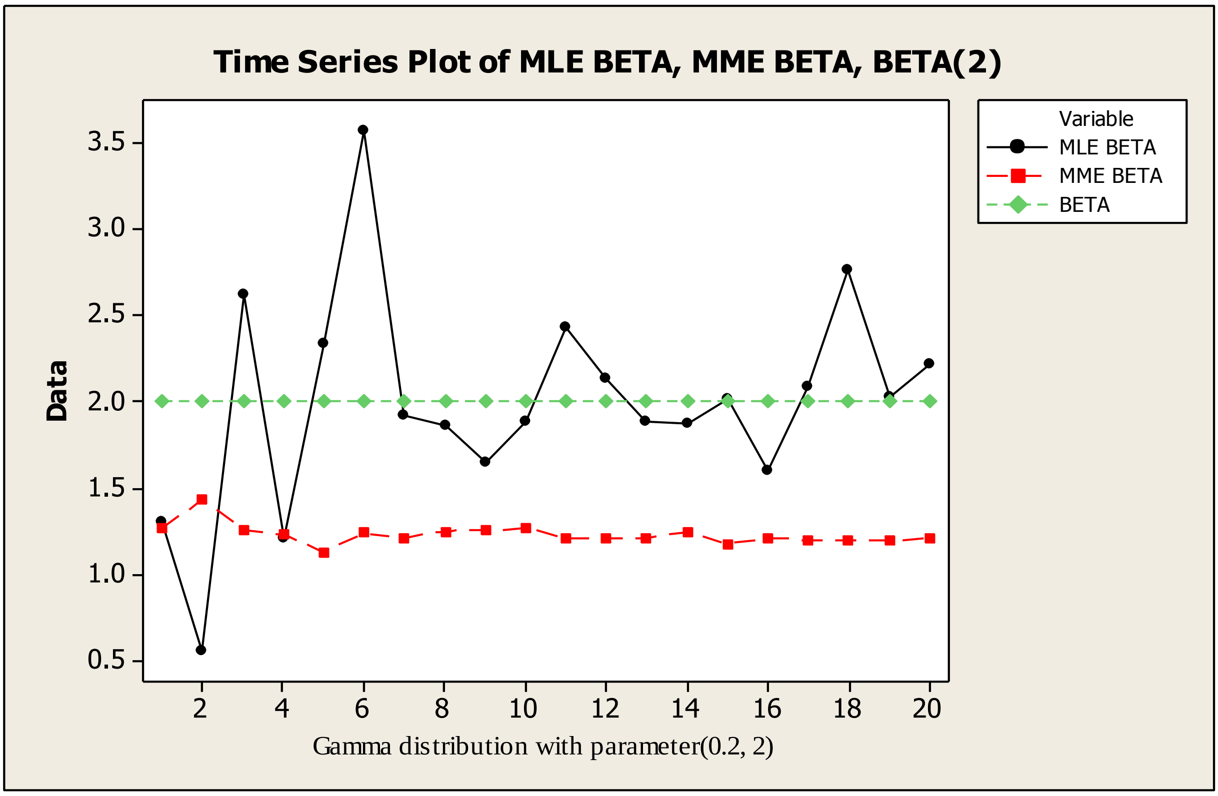

Figure 8. Gamma distribution with parameters [0.2, 2] by the cost of living data

Table 5

We assumed that and [$(\beta = 0.2)$, $(\alpha = 2)$]

N	MME ALFA	MLE ALFA	MME BETA	MLE BETA
10	2.17856	2.486478	3.17856	0.119437
20	2.54659	2.137833	3.54659	0.159778
30	2.28883	1.738335	3.28883	0.223157
40	1.89792	1.786918	2.89792	0.230021
50	2.29019	2.138279	3.29019	0.205509
100	1.50626	1.734277	2.50626	0.225345
150	1.97360	2.213167	2.97360	0.170811
200	1.75733	2.504296	2.75733	0.173530
250	1.91382	1.983903	2.91382	0.213967
300	1.98827	1.926341	2.98827	0.209494
350	1.73233	1.988568	2.73233	0.189888
400	2.17473	2.117814	3.17473	0.193756
450	2.02141	2.114542	3.02141	0.187307
500	1.95328	1.850457	2.95328	0.129193
550	2.02401	1.977301	3.02401	0.198306
600	1.99521	1.813609	2.99521	0.221386
650	2.05348	2.058845	3.05348	0.199091
700	2.15821	2.163662	3.15821	0.186329
750	2.17180	2.097983	3.17180	0.196375
1000	2.17180	2.127847	2.87020	0.184653

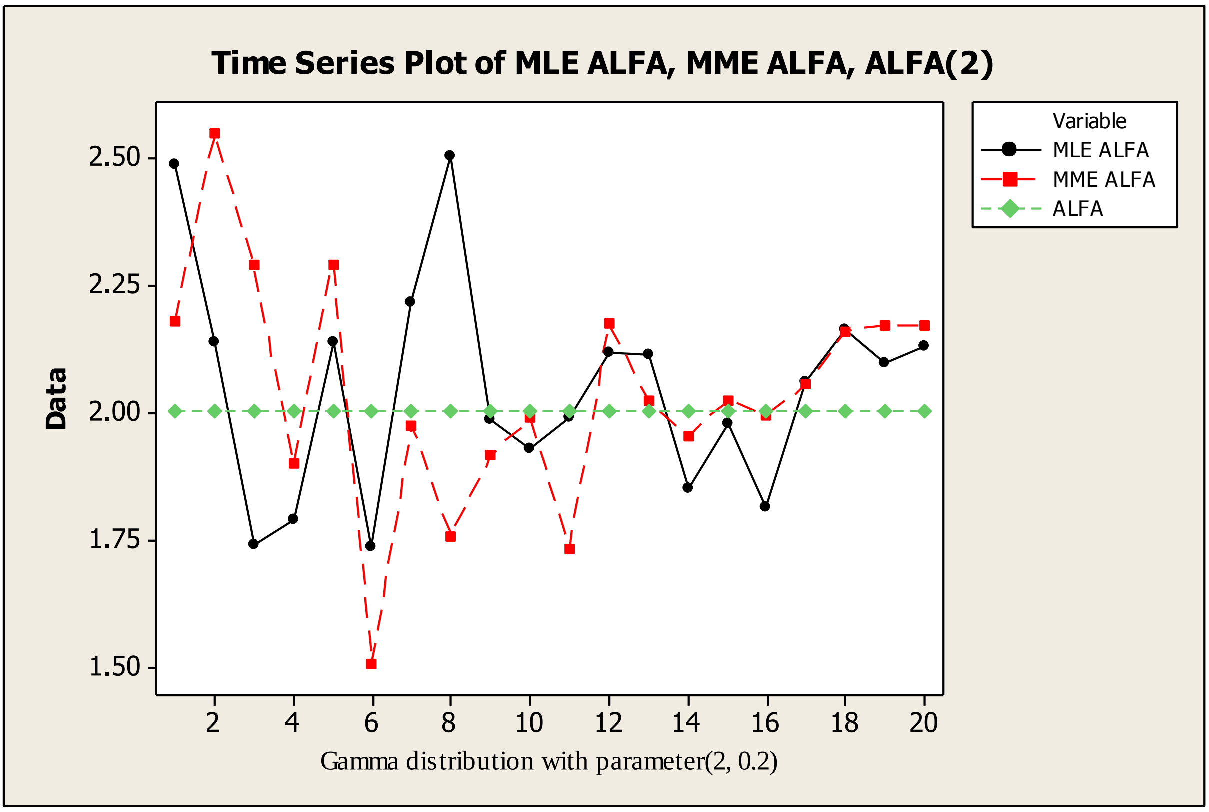

Figure 9. Gamma distribution with parameters [2, 0.2] by the cost of living data

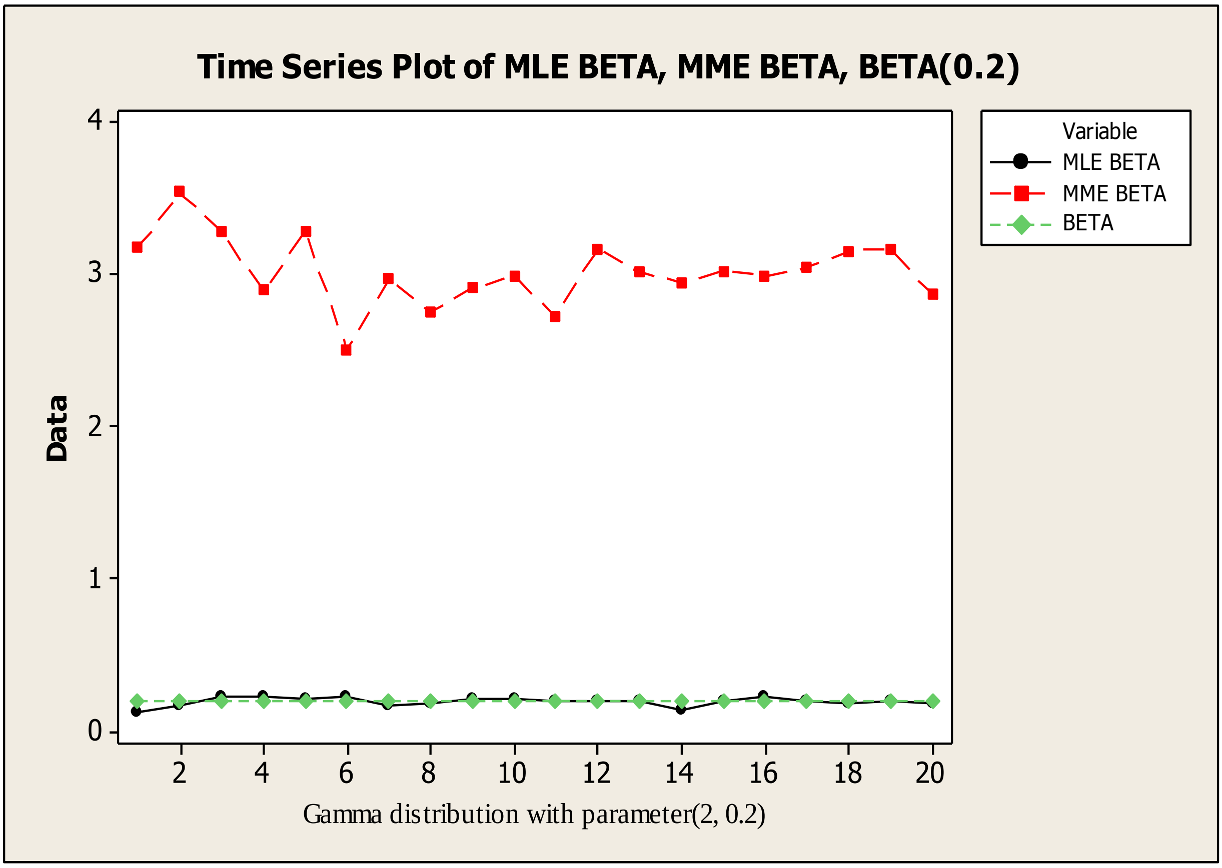

Figure 10: Gamma distribution with parameters [2, 0.2] by the cost of living data

By using the cost of living data The purpose of studying estimation theory is to arrive at an estimator, which is preferably practicable and usable. The estimator takes the measured data as input and produces an estimate for the parameters. Preferred also is an estimator with an exemplary character. Estimator idealism usually refers to achieving a minimum average error over a range of some class of estimators, for example, an unbiased estimator with the least variance. In this case, the class is a set of unbiased estimators, and the mean error value is the variance value (the mean squared error between the estimated value and the parameter). However, optimum capabilities do not always exist. To arrive at the required estimate, it is necessary first to determine the probability distribution of the measured data, and the extent to which the distribution relates to the relevant unknown parameters. Often, the probability distribution may be derived from physical models that clearly show the relationship of the measured data to the parameters to be estimated, and how the data is corrupted by random errors or noise. In other cases, the probability distribution of the measured data is simply "assumed", for example, based on familiarity with the nature of the measured data and/or analytical suitability.

After making a decision based on a probabilistic model, it is helpful to find the limitations of the estimator. This limitation, for example, can be found through the "Cramér – Rao" bound.

Hence, the estimator needs to be developed or implemented if a known and valid estimator exists for this model. The estimator must be tested against the constraints to determine if it is the optimal estimator (and if so, no other estimator will perform better than it).

Finally, simulation experiments can be run and conducted using the estimator to test their performance.

After the estimator is reached, the real data can show that the model used to arrive at the estimator is incorrect, which may require repeating these steps to find a new estimator.

An estimate that is not feasible or feasible can be revoked and the process can be started over.

In summary, the estimator estimates the parameters of a physical model based on the measured data.

Conclusions. In this paper, we consider two methods of Estimate Moment likelihood method MME and Maximum likelihood method MLE, and The results of the estimate the population parameters were as follows: generated random samples with different sizes of 10, 20, 30, 40, 50, 100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600, 650, 700, 750, 1000, from the gamma distribution with the parameters of the distribution we estimated each time, and after performing the statistical analysis process by the method of Maximum (MLE) and method of moments (MME) to estimate whether the results of the estimation process approach the values of the imposed parameters. We found that the maximum likelihood method (MLE) gives results that are close to the original distribution parameters from the moment method (MME), and this has been proven in the practical study, and we recommend that follow the Maximum likelihood method to estimate the parameters of the statistical population.

References:

1. Al-Bouzidi, A, (2000), Principles of Statistical Inference. Al-Fateh University Publications.
2. Ali, A.,& Ali, A. (2020), Statistics and Theoretical Probabilities and Practice. ELGA Publications.
3. G.P.BEAUMONT, Intermediate Mathematical Statistics.
4. Norman, L. Johnson (2020), continuous univariate distributions.
5. Data from Libyan ministry of oil and gas.

Стаття надійшла до редакції 07.12.2020 р.